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Tsallis' Analysis of the Horizontal Component of the Earth's Magnetic Field over India during 2002

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Authors' contributions

This work was carried out in collaboration among all authors. All authors read and approved the final manuscript.

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ABSTRACT

A widely used measure of entropy to quantify uncertainity in an open system is the Boltzmann-Gibbs (B-G) entropy. It, however, cannot describe non-equilibrium systems with large variability and multi-fractality. A generalisation of the B - G entropy is the Tsallis' entropy. The values of the horizontal components of the Earth's magnetic field, observed at various stations in India in 2002 were used. During the years 2000 – 2002, solar cycle 23 reached its maximum. Data from Ettaiyapuram (ETT, latitude = 9^0 10' N, longitude = 78^0 01' E, geomagnetic latitude = 0.13^0 N), Visakhapatnam (VIS, 17^0 41' N, 83^0 19' E, 8.17^0 N), Hyderabad (HYD, 17^0 25' N, 78^0 33' E, 8.17^0 N), Alibag (ABG, 18^0 37' N, 72^0 52' E, 10.02^0 N) and Tirunelveli (TVI, 8^0 42' N, 77^0 48' E, 0.32^0 S)

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were used. Using these values as inputs, we demonstrate that Tsallis' entropy can be used to detect minor differences in the horizontal components of the geomagnetic field observed between different pairs of stations. The method is a very simple and elegant one to detect minor variations between pairs of similar signals.

Keywords: Tsallis' entropy; horizontal component of the geomagnetic field; detection of dis-similar signals.

1. INTRODUCTION

Geomagnetic time series are often generated by complex spatio-temporal dynamics of which nonlinearity and scaling are the most important processes. It is well known that the main geomagnetic field variations originate inside the Earth while its short term fluctuations are due to external sources. While the solar daily variation is a fairly regular process, the irregular fluctuation (the disturbance component of the geomagnetic field) is a remarkably nonlinear process. Thus, from the point of view of space weather, the complete analysis of irregular and intense geomagnetic variations are relevant due to the possible solar-geomagnetic coupling adverse effects on power lines and data transmission by satellites [1].

A prominent global phenomenon that inter-links the solar wind, magnetosphere, ionosphere, atmosphere and the Earth's surface is a magnetic storm. A storm is an interval of time when a sufficiently intense and long-lasting convection electric field leads, through a substantial energization in the magnetosphere-ionosphere system, to an intensified ring current strong enough to cause the irregular fluctuations mentioned above. Magnetic storms are quantified by D_{st} values; other indices used for quantifying them being the Ap, Kp, EEJ, etc. indices.

In a pioneering study Balasis et. al. used Tsallis analysis for analyzing a D_{st} time series [2]. A D_{st} (Disturbance Storm Time) is a measure of the Earth's geomagnetic activity and is widely used to characterize a geomagnetic storm. They showed that Tsallis' entropy can effectively detect the dissimilarity of complexity between pre-storm activity and intense magnetic storms. This study was extended, using one year (2001) of D_{st} data with special emphasis on application to the Earth's magnetosphere [3]. Another study which was narrowed down to two intense magnetic storms was also carried out [4]. Their analyses showed the dissimilarities among different physiological (quiet-time) and pathological (intense magnetic storm) states of the magnetosphere. The ability of Tsallis entropy analysis to distinguish between dis-similar states was thus well established.

Since these pioneering studies, Tsallis' entropy analysis has been applied to a wide variety of areas: to space and solar wind plasmas [5,6], solar flares [7], magnetic storms and magnetospheric dynamics [7–9], Earth's climate [10], aerosols [11], econophysics [12], geomagnetically induced currents [13], seismic signals [14] and engineering [15].

To explore whether Tsallis' entropy analysis can detect minor variations between similar signals, we carried out a Tsallis analysis of the horizontal component of the geomagnetic field observed at different locations in India during 2002; during the years 2000 – 2002 solar cycle 23 reached its maximum. Our results show that Tsallis' entropy analysis can indeed differentiate even minor dissimilarities in the horizontal component of the geomagnetic field between different pairs of stations.

2. MATERIALS AND METHODS

2.1 Principle of Tsallis' Entropy

In this section we outline the principles of Tsallis' entropy; more elaborate accounts can be found elsewhere [2, 4]. The Boltzmann-Gibbs (B - G) entropy, which is the widest known measure of uncertainity in statistical mechanics, is used to quantify the uncertainity of an open system. This entropy, however, cannot describe non-equilibrium systems with large variability and multi-fractal structures. Tsallis, therefore, proposed a generalisation of the Boltzmann-Gibbs statistics with an entropy function characterized by an index q and which is given by [16]

$$S_q = k_B \frac{1}{q-1} \left(1 - \sum_{i=1}^{W} P_i^{\,q} \right) \qquad (1)$$

In (1), P_i are the probabilities associated with the microscopic configurations, W is their total number, q is a real number and k_B, the Boltzmann's constant. The value of q is a measure of the non - extensivity of the system; q → 1 corresponds to the standard extensive BG statistics.

We estimate S_q based on the concept of symbolic dynamics [17,18] and the technique of lumping [2]. Exemplifying, a threshold T is chosen (usually the mean of the data being considered) and the symbols of "1" and "0" are assigned to the signal depending on whether it is above or below T. This binary partition will generate a symbolic time series from a 2 letter (λ = 2) alphabet (0, 1) a sequence of the form 01101001100110……….Reading this sequence by lumping of length L = 2, we obtain 01/ 10/ 10/ 01/ 10/ 01/ 10……The number of all possible kinds of blocks is $\lambda^L = 2^2 = 4$, namely 00/ 01/ 10/ 11. Thus the required probabilities for the estimation of Tsallis' entropy P_{00}, P_{01}, P_{10}, P_{11} are the fractions of the blocks 00, 01, 10, and 11 in the symbolic time series.

The Tsallis' entropy S_q for a word length L is therefore

$$S_q(L) = k_B \frac{1}{q-1}\left(1 - \sum_{A_1, A_2, \ldots A_L} [P^{(L)}_{A_1, A_2, \ldots, A_L}]^q\right) \tag{2}$$

2.2 Data Analysis

As mentioned in section 1, the aim of this study is to examine whether Tsallis' entropy analysis can be used to detect minor variations in similar signals. The horizontal component of the geomagnetic field, recorded at 1 hr intervals, at Ettaiyapuram (ETT, latitude = 9^0 10' N, longitude = 78^0 01' E, geomagnetic latitude = 0.13^0 N) (Visakhapatnam (VIS, 17^0 41' N, 83^0 19' E, 8.17^0 N), Hyderabad (HYD,17^0 25' N, 78^0 33' E, 8.17^0 N), Alibag (ABG, 18^0 37' N, 72^0 52' E, 10.02^0 N) and Tirunelveli (TVI, 8^0 42' N, 77^0 48' E, 0.32^0 S) were used for the study (https://www.iigm.res.in).

One way to examine for a transient phenomenon is to divide the time series into shorter time intervals and then analyze these time windows separately. If this analysis yields different results for corresponding windows; for instance, if the result for a time window containing a transient from one station is different from the result of the same window from the other station, then the transient that occurred at the first station can be identified. This was the principle used to differentiate the regular state of the magnetosphere from a state where an intense magnetic storm had occurred [2, 3].

The time series associated with the horizontal (H) component of the Earth's magnetic field was divided in to five intervals. The average of H for each time interval was then determined as $\bar{H}$ The symbolic series was then generated by assigning a zero (or one) if the hourly values of H was less (or greater) than $\bar{H}$. The symbolic series so generated was used to calculate the Tsallis' entropy given in (2).

3. RESULTS AND DISCUSSION

We now discuss the results obtained by Tsallis' entropy analysis of the time series of the horizontal component of the geomagnetic field recorded at various locations in India during 2002. The entropies shown have been normalized with respect to the entropies given in (2) for a uniform distribution of probabilities. In all the figures, the X- axis denotes the days while the Y-axis has the H component. A dis-similarity that occurred at any station is identified by an "arrow", while the time intervals (on the X-axis) are demarcated by an inverted V (i.e., Λ).

Fig. 1 exhibits the recordings of the H-field recorded at Hyderabad (HYD; 17^0 25' N, 78^0 33' E, 8.17^0 N) and Alibag (ABG; 18^0 37' N, 72^0 52' E, 10.02^0 N) versus time in days. The Tsallis' entropy calculated for q = 1.2 and 1.5 is shown in the lower part of the figure. During the interval of 1 – 75 days (window 1 or W1), a stronger fluctuation occurred at HYD (indicated by an arrow) as compared to the recording at ABG. The Tsallis' entropy of HYD is thus greater than the entropy at ABG for q = 1.2. During the second and third windows (W2 and W3, extending from 76 – 125 days and 126 - 265 days respectively) stronger fluctuations occurred at ABG; the Tsallis entropy for ABG is therefore greater. For W4 and W5 (extending from 265 - 285 days and from 286 – 365 days respectively), the H–components are similar and therefore the entropies are equal. The conclusions for q =1.5 are the same as the conclusions for q = 1.2.

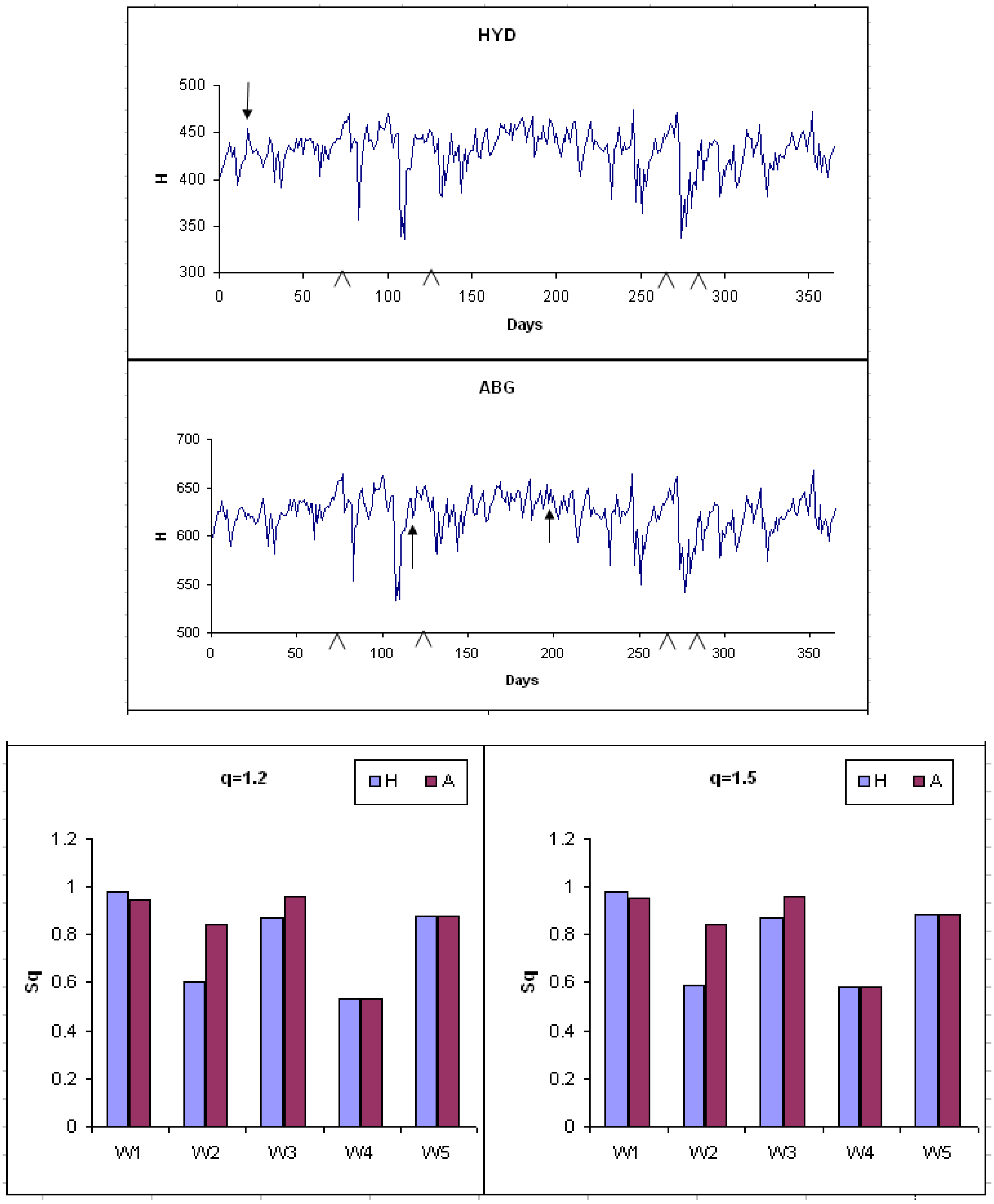

Fig. 1. The upper and the middle panel depicts the horizontal component of the geomagnetic field observed at Hyderabad (HYD) and Alibag (ABG) versus time (in days). The arrows indicate the days when the signals were not similar. The panel at the bottom depicts the Tsallis' entropy for q = 1.2 and 1.5 for the five time windows into which the signals were divided

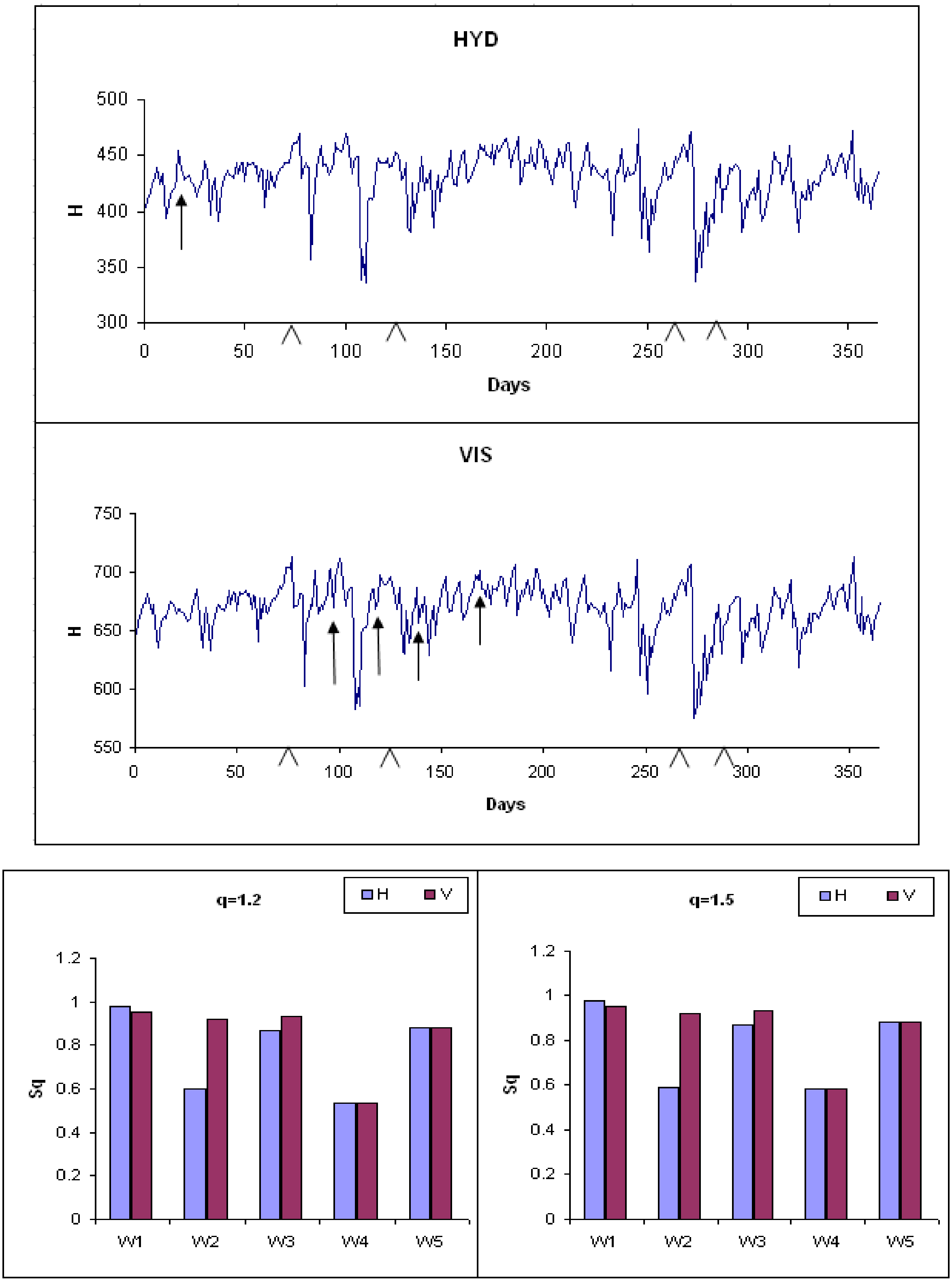

Fig. 2. Horizontal component of the geomagnetic field recorded at HYD and Visakhapatnam (VIS) is depicted as a function of time (in days). The arrows again indicate the days where the fields were not similar. The bottom panel shows the Tsallis' entropy for q = 1.2 and 1.5 for the five time windows

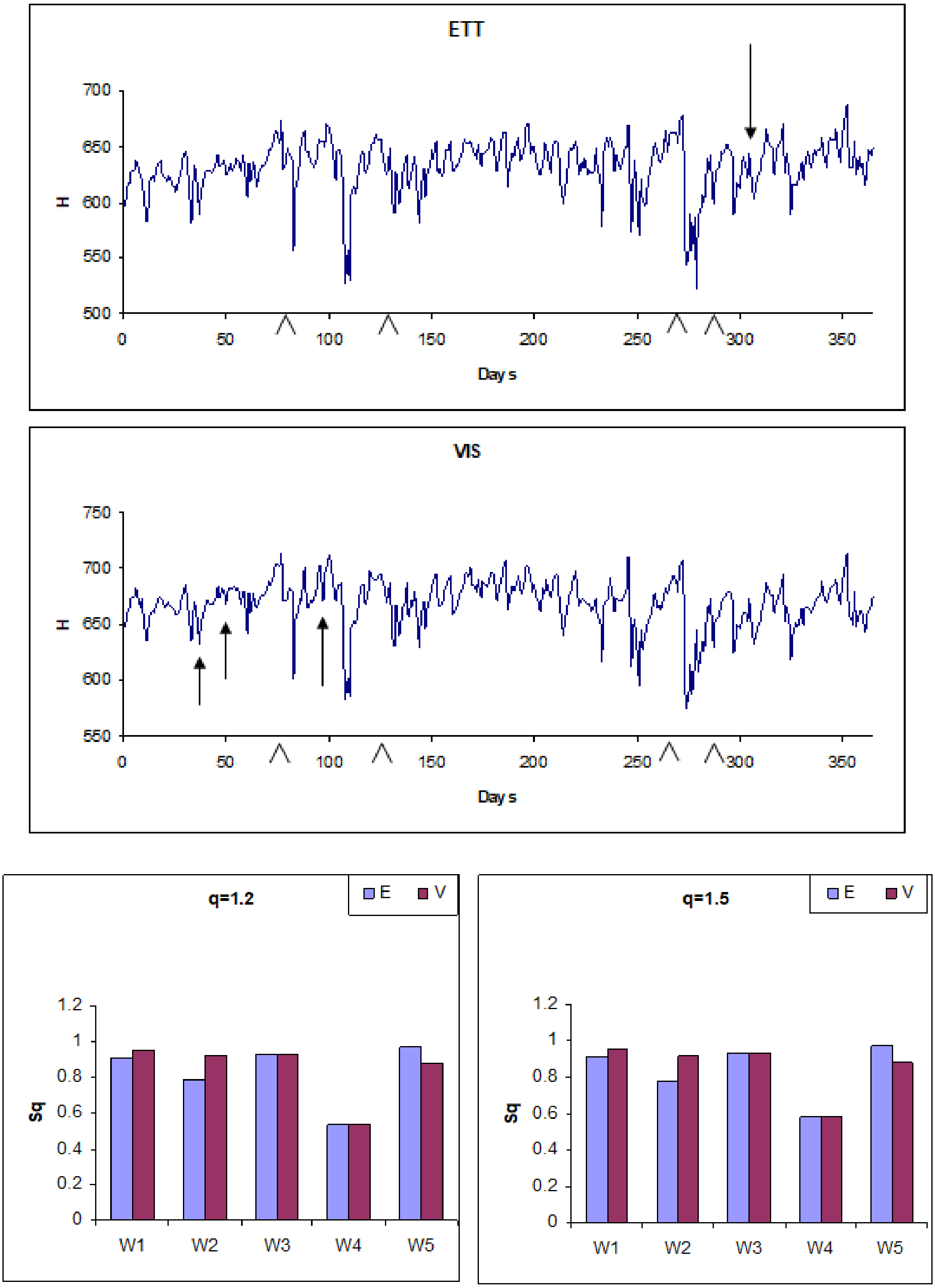

Fig. 3. Horizontal component of the geomagnetic field as a function of time (in days) is depicted for Ettaiyapuram (ETT) and VIS. Differences in the components are indicated by arrows. The bottom panel depicts the Tsallis' entropy for q = 1.2 and 1.5 for the five time windows

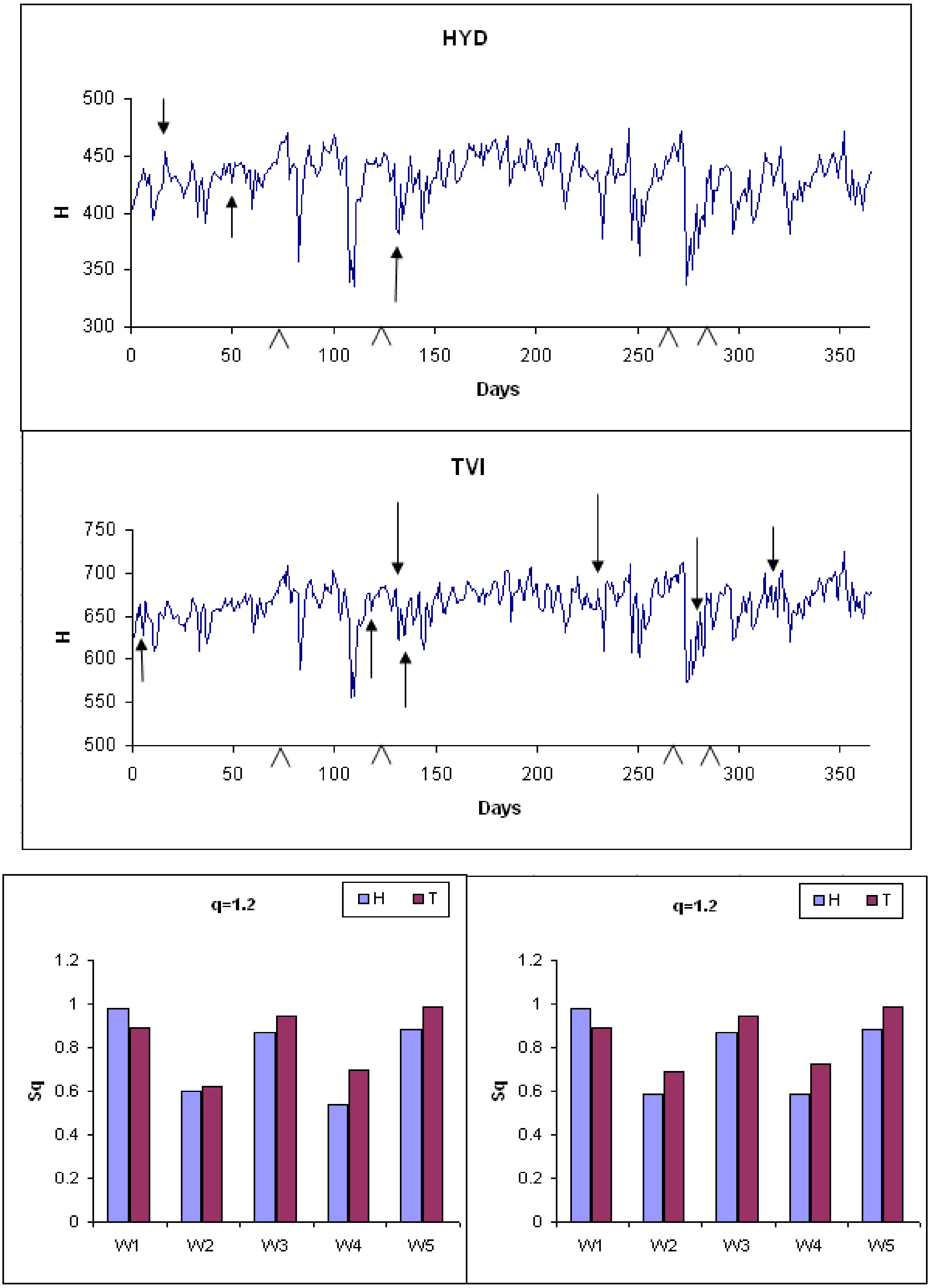

Fig. 4. The horizontal component of the geomagnetic field observed at HYD and Tirunelveli (TVI) as a function of time (in days) is depicted. The differences in the two components are again indicated by arrows. Tsallis' entropy for q = 1.2 and 1.5 is depicted in the lower panel

Fig. 2, which is similar to Fig. 1, is interesting for an additional reason: both Hyderabad (HYD; 17^0 25' N, 78^0 33' E, 8.17^0 N), and Visakhapatnam (VIS, 17^0 41' N, 83^0 19' E,8.17^0 N) have the same latitude. W1 for HYD has a stronger fluctuation leading to a larger Tsallis' entropy as compared to the Tsallis' entropy for VIS. W2 for VIS has two differences as compared to HYD, leading to a higher S_q for VIS as compared to HYD. Again, in W3, the stronger fluctuation is at VIS which consequently has a higher S_q. The signals are similar in W4 and W5, leading to the same S_q. Again, S_q for q = 1.5 is consistent with S_q for q = 1.2.

Fig. 3 exhibits the Tsallis' entropy variation between Ettaiyapuram (ETT, 9^0 10' N, 78^0 01' E, 0.13^0 N) and VIS (17^0 41' N, 83^0 19' E, 8.17^0 N). In windows W1 and W2 the stronger variations are at VIS, giving it a higher value for Tsallis' entropy. In W3 and W4, the H- fields are similar thus S_q is the same, while in W5 the stronger signal is at ETT giving it a larger Tsallis' entropy. Again, S_q for q = 1.5 is consistent with the Tsallis' entropy for q = 1.2.

Finally, Fig. 4 compares the Tsallis' entropy variation between HYD (17^0 25' N, 78^0 33' E, 8.17^0 N) and Tirunelveli (TVI, 8^0 42' N, 77^0 48' E, 0.32^0 S). For W1, HYD has two dis-similarities as compared to one of TVI; the Tsallis' entropy for HYD is therefore larger. In W2, TVI has only one smaller dis-similar signal; consistent with our results, S_q is larger for TVI as compared to that of HYD. For the other windows, namely W3, W4 and W5 stronger dis-similar signals occur at TVI; Tsallis' entropy in these windows is thus larger for TVI as compared to that of HYD. Again, the conclusions for q = 1.5 are consistent with the results for q = 1.2.

4. CONCLUSIONS

In a Harmonic Analysis study of the geomagnetic field at an equatorial station, it was shown that terms up to order five can adequately model the observed geomagnetic field. The 24-hour periodicity was also confirmed [19]. A complementary wavelet based semblance analysis discovered an additional 12-hour periodicity [20]. In both these studies only macro-variations of the signals were detected.

In contrast we have, in this study, applied the concept of Tsallis' entropy to the horizontal component of the Earth's magnetic field observed at different observation stations in India during 2002; the solar cycle 23 was at its maximum during the years 2000 – 2002. Dividing the data into different time windows (in our case, five) and comparing the Tsallis' entropy between corresponding windows between pair of stations we have conclusively shown that this method can detect dis-similarities between pairs of signals. This is also in contrast to the studies in [2–4] where complexity was studied in a single set of data.

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COMPETING INTERESTS

Authors have declared that no competing interests exist.

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